Review

under the procedure for public defence of the PhD Thesis entitled:
"Toroidal compactifications of discrete quotients of the complex two-ball"
under the procedure for acquisition of the educational and scientific degree "Doctor"
by Pancho Georgiev Beshkov

In the Scientific field: 4. Natural Sciences, Mathematics and Informatics,
Professional field: 4.5. Mathematics,
Doctoral program: "Algebra, Topology and Applications", Department: "Algebra",
Faculty of Mathematics and Informatics (FMI),
Sofia University "St. Kliment Ohridski" (SU).

This Review has been prepared by prof. Peter Boyvalenkov, Dr.Sci., Institute of Mathematics and Informatics, BAS, in my capacity as a member of the scientific jury, according to Order for the defence of this PhD thesis according to Order RD-38-141/04.03.2022 of the Rector of the Sofia University "St. Kliment Ohridski" and decision of the scientific jury (Protocol 1/14.03.2022).

1 General characteristics of the dissertation thesis and the presented materials

The presented PhD thesis is in Bulgarian, 126 pages long and consists of an introduction, four chapters and a bibliography of 50 titles. The abstract, (19 pages), presents in short the content of the dissertation and clearly and accurately reflects both the main contributions of the doctoral student and approbation of the results. The dissertation is based on results published in two papers of the doctoral student, one in the Annual of Sofia University, FMI, (2019) and the other in the Proceedings of the Bulgarian Academy of Sciences (2021). These achievements correspond to the commonly accepted requirements for a meaningful dissertation, in particular to the minimal requirements of Art. 2b. para. 2 and para. 3 of the Act on Development of the Academic Staff in the Republic of Bulgaria. The complete set of the remaining required documents is presented as well.

2 Short CV and personal impressions of the candidate

I have known Mr. Beshkov briefly, only a a listener of his talks. Despite that, I would like to note that he knows the material and presents it confidently. The documents presented show that Mr. Beshkov was a part-time lecturer, a doctoral student, and now an assistant at the department Algebra in FMU-SU. He has acquired a Bachelor's degree, major Mathematics, from FMI-SU, in 2010, a Master's degree again from FMI-SU in the master's program " Mathematics and mathematical physics ". He he has been enrolled as a full-time PhD student in the doctoral program "Algebra, Topology and Applications", Department of "Algebra", FMI-SU at 15.02.2017. He completed his PhD study with the right to defend, as of 15.02.2020, by Order RD-20-401/06.02.2020.

3 Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure

The present PhD thesis investigates the smooth toroidal compactifications $X = (B/\Gamma)'$ of non-compact quotients B/Γ of the complex two-ball B by a sublattice Γ of $U(1,2)$.

The first two chapters introduced the necessary preliminaries – definitions and known results. Holomorphic vector bundles over a complex manifold, algebraic surfaces, and their Chern numbers are considered in Chapter 1 and the logarithmic Bogomolov-Miyaoka-Yau equality, characterizing the smooth toroidal compactifications $(B/\Gamma)'$ of ball quotients B/Γ is derived in Chapter 2.

The results of the doctoral student are presented in Chapters 3 and 4. Chapter 3 is based on the paper [6] (the references' numbers are taken from the dissertation). It establishes and investigates an explicit bijective correspondence between the unramified coverings $X_1 = (B/\Gamma_1)' \to X = (B/\Gamma)'$ of a smooth toroidal compactification $X = (B/\Gamma)'$ and the finite unramified coverings $Y_1 \to Y$ of a minimal model Y of X. In Chapter 4, based on the publication [7], the author obtains lower bounds on the number k of the cusps of B/Γ, using the logarithmic Bogomolov-Miyaoka-Yau equality for (X, D). In both cases, derivation of the results requires significant amount of related investigations. The results presented allow me to conclude that the candidate Pancho Georgiev Beshkov has in-depth knowledge of the PhD thesis, and that his original

contributions are sufficient to acquire the educational and scientific degree "Doctor".

4 Approbation of the results

The dissertation is based on results published in two papers – in Annual of SU, FMI (2019) and Proceedings of BAS (2021), respectively, the second with impact factor 0.378 for 2020. Both publications are co-authored with the supervisor Prof. Azniv Kasparian, and the first also with G. Sankaran. Declarations for co-authorship are presented to show that the contribution of all authors is equal. The results were presented by the doctoral student at the Spring Scientific Session of the FMI in 2019 and 2021 and at the National Seminar on Coding Theory "Professor Stefan Dodunekov" in 2018 and 2019, respectively. I do not have information about citations to the publications.

5 Qualities of the abstract

The abstract in Bulgarian is 20 pages (in English – 19 pages), it is prepared according to the requirements and correctly reflects the content of the dissertation and scientific contributions of the doctoral student.

6 Critical notes and recommendations

The presentation could be in more detail and defining at least the basic notions and with example. The style chosen makes the reading difficult and makes the dissertation almost impossible for reading by students and beginning doctoral students. There are still some technical (grammar) mistakes. There remarks do not change my overall good impression for the dissertation and the work of the doctoral student.

7 Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, I confirm that the presented PhD thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, fulfils the requirements of the ADAS in the Republic of Bulgaria, the Rules for its Implementation and the corresponding Rules

at the Sofia University "St. Kliment Ohridski" (FMI-SU) for acquisition by the candidate of educational and scientific degree "Doctor" in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5. Mathematics. In particular, the candidate meets the minimal national requirements in the professional field and no plagiarism has been detected in the scientific papers submitted for the competition.

Based on the above, I recommend to the scientific jury to award to Pancho Georgiev Beshkov the educational and scientific degree "Doctor" in the Scientific field: 4. Natural Sciences, Mathematics and Informatics, Professional field: 4.5. Mathematics, Doctoral program: "Algebra, Topology and Applications", Department of "Algebra", Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski".

May 10, 2022
Sofia

Signature:
Prof. Peter Boyvalenkov, DrSci