REVIEW

under the procedure for acquisition of the educational and scientific degree "Doctor"
by candidate Boyan Kolev Stefanov**,**
of the PhD Thesis entitled: "Optimal Control Problems under Uncertainty",
In the Scientific field: **4. Natural Sciences, Mathematics and Informatics**
Professional field: **4.5. Mathematics,** Doctoral program **Operations Research**,
Faculty of Mathematics and Informatics (FMI), **Sofia University "St. Kliment Ohridski"**

The review has been prepared by: **Prof. Dr. Vladimir Veliov, Vienna University of Technology**, as a member of the scientific jury for the defense of this PhD thesis according to Order № РД-38-129/1.3.2024 of the Rector of the Sofia University.

This review follows the structure and the specific points suggested in compliance with the rules and regulation of the Sofia University "St. Kliment Ohridski".

1. General characteristics of the dissertation thesis and the presented materials

The presented materials have been prepared in accordance with the Law on the development of the Academic Staff in the Republic of Bulgaria (LDASRB), the Regulations for the application of the LDASRB, as well as the Regulations for the Terms and Procedures for Acquiring Scientific Degrees and Holding Academic Positions (RT LDASRB) in SU" St. Cl. Ohridski". These include: PhD Thesis in English; Abstract in Bulgarian; Autobiography in Bulgarian; List of scientific publications on the topic of the dissertation; Reference on the fulfillment of the minimum requirements under Article 2b, para. 2 and 3 of the LDAS of the Republic of Bulgaria; Application by the PhD student; Doctoral student's declaration of originality and absence of plagiarism; Report from the research supervisor; Similarity report; StrikePlagiarism.com software product report for no plagiarism; Protocol for verification of originality; Statement regarding the anti-plagiarism procedure signed by the supervisor and others.

The dissertation contains 72 pages and consists of introduction and 3 more chapters presenting the author's contributions, and a short concluding chapter. The bibliography contains 53 items.

2. Personal impressions of the candidate

I know Bayan since about three years ago, and in all this time I have been kept informed about his work. I have attended several of his seminar and conference talks and have a very good impression about the quality of his presentations. My general impression is that Boyan is a knowledgeable and skillful mathematician and a hard and dedicated worker. His research has often been a subject of discussion between his scientific advisor, Prof. M. Krastanov and myself, therefore I have a good impression about the progress of his work.

3. Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure

In Chapter 1 the author considers an infinite horizon continuous time linear-quadratic differential game with constrained controls, where the constraining set contains the origin in its interior. Since the solution of the unconstrained problem on an infinite horizon is known from the literature (although the author presents detailed proofs, which is to be appreciated), and the solution of the constrained problem on a finite interval is easier to handle with, the author develops

the following idea: (i) to obtain conditions under which the trajectories of the system converge to zero for any admissible controls (in particular for those resulting from the constrained game on a finite horizon) and to implement the corresponding feedbacks; (ii) then to implement the unconstrained linear feedback controls on the remaining infinite horizon, which automatically satisfy the control constraints if a sufficiently small neighbourhood is attained in the first phase (thanks to the assumption that the constraint contains the origin in its interior). This separates the two difficulties: infinite horizon and constrained control. The idea is nice, the proofs are correct, although the formulation of the results is misleading, as it will be explained in Point 6 below.

Similarly to Chapter 1, in Chapter 2 the author considers an infinite horizon linear-quadratic dynamic game with constrained controls, where the constraining set contains the origin in its interior. This time the dynamics is represented in discrete time, which makes the required technique different. The basic idea is similar to that in Chapter 1 -- to decompose the problem in two phases, one on a finite horizon with control constraints, and a second one on an infinite horizon without control constraints. Ultimately, a pair of feedback controls is constructed that solves a meaningful game, which, however, is not correctly described (see my comments in point 6 below). A practically meaningful example is numerically analysed without enough details presented in the thesis.

Chapter 3 is devoted to Pontryagin-type conditions for infinite horizon discrete time optimal control or game problems. The results are based on a paper by Aseev at al., 2017, where necessary optimality conditions were presented with the corresponding adjoint variable being explicitly defined (without involvement of transversality or asymptotic conditions at infinity). There are two results in this chapter. The first claims that, on convexity assumptions, the necessary optimality condition with the explicit adjoint variable is sufficient for weekly overtaking optimality. In the second result (which is first in the exposition of the thesis), the notion of weakly overtaking Nash equilibrium is defined in a natural way, and a system of conditions is obtained, which are necessarily satisfied. Both results are to be expected, but the derivation requires good technical skills. An example is given, where the Nash equilibrium is obtained explicitly by utilization of the necessary conditions.

4. Approbation of the results

The results presented in the dissertation are published or submitted in 4 papers: two of them are published, one is accepted, and one is submitted. Two of the papers are book chapters, one
appeared in the reputable journal *Dynamic Games and Applications.* The results are recently published and have a small visible impact so far. However, they contain new results and bring productive ideas, therefore an impact of the research in this area is to be expected.

In my opinion, the dissertation has the potential to meet (see Remark 1 below) the national and the additional requirements of Sofia University “St. Kliment Ohridski” for acquiring the educational and scientific degree “Doctor” in the scientific field and professional field of the procedure. The presented results do not repeat such from previous procedures for acquiring a scientific title and academic position. I do not see any plagiarism. As far as I know from Prof. Krastanov (who is a co-author of the papers) Boyan has done a substantial creative work on all the papers.

The scientific metrics of these articles, compared with the minimum requirements for the educational and scientific degree "doctor", according to Resolution No. 26 of February 13, 2019 on the amendment and addition of the Regulations for the implementation of the Law on the development of the academic staff in the Republic of Bulgaria, adopted with Decree No. 202 of the Council of Ministers of 2010 (promulgated, SG No. 75 of 2010; amended and supplemented, No. 19 of 2011, No. 9 of 2012, No. 62 of 2013, No. 60 of 2014, No. 57 of 2015 and No. 56 of 2018) are as follows: All publications fall into **Group G7** and collect a **total of 105 points,** with a minimum requirement of **30 points** for the acquisition of the educational and scientific degree "doctor" in the scientific field **4. Natural sciences, mathematics and informatics**, professional direction **4.5 Mathematics**. The first publication is in Q3 and is valued at 45 points, the second is with SJR and is valued at 30 points, and the third is with SJR and is valued at 30 points. The obtained **105 points** significantly exceed the minimum requirements for obtaining the educational and scientific degree "doctor" in the scientific field and professional direction of the procedure.

5. Qualities of the abstract

The abstract correctly and comprehensively presents the content of the thesis and is well written. At this place, I mention that the title of the dissertation is too general and does not give accurate information about the subject and the content.

6. Critical notes and recommendations

The formulations of the results in Chapter 1 do not correspond to what is proved. The author speaks about an open-loop min-max game (with the first player as a leader) and the value of the game is defined correspondingly. However, Proposition 1.2 and hence, the main result in the chapter -- Proposition 1.7 -- concern the game in linear feedback controls with complete and exact state information. The results are meaningful, but the formulations are wrong. For example, the concluding equality in Proposition 1.7 is true, but with a value function defined in a rather different way than in (1.2). The same applies to the formulation of Corollary 1.9.

The situation is similar in Chapter 2. In Theorem 2.5, for example, the crucial statement, namely that "min max = max min" is correct, but the equality to V(x_0, k_0) is wrong. The formulation of Theorem 2.8 also requires an appropriate correction. The author just has to say what is actually proved. Theorem 2.1 is fundamentally wrong. This reviewer has sent to the advisor of Boyan a counterexample. Again, the reason is in mixing up various kinds of control strategies. Notice, that Theorem 2.1 is used in Section 2.3.

Some minor remarks follow.
There are two mistakes in Definition 0.1.
The space l^2(N_{k_0},R^m) on page 17 is actually independent of k_0!
What is U_k on page 17?
After (2.2), "first player" should be "second player".

Chapter 3 is badly organized, which leads to repetitions: the control case had to be placed before
the game.

I have two additional questions. The first one concerns the idea developed in chapters 1 and 2. Usually in economics the infinite horizon problems of interest have discounted integrant. Can a similar idea be applied to such problems? The second question is how do you solve the constrained game on the finite horizon interval.

Remark 1. The present review is based on the official submission of the thesis. There are substantial inaccuracies and mistakes, however, I think that they all are avoidable and can be

corrected. My recommendation stated below may change depending on whether Boyan would give a convincing argument in a supplementary corrected version (annex to the thesis) and during the defence.

7. Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I conditionally confirm (see Remark 1 above)** that the presented PhD thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the LDASRB in the Republic of Bulgaria, the Rules for its Implementation and the corresponding Rules at the Sofia University "St. Kliment Ohridski" (FMI-SU) for acquisition by the candidate of educational and scientific degree "Doctor" in the Scientific field **4. Natural Sciences, Mathematics and Informatics**, Professional field **4.5. Mathematics**, doctoral program **Operations Research**. In particular, the candidate meets the national requirements in the professional field and no plagiarism has been detected in the published scientific papers.

Based on the quality and the importance of the obtained scientific results **I conditionally recommend (see Remark 1 above)** to the scientific jury to award Bayan Kolev Stefanov the educational and scientific degree "Doctor" in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5 Mathematics.

08.05.2024 Reviewer:

Prof. Dr. Vladimir Veliov