Review

on the PhD Thesis of Boyan Kolev Stefanov titled

Optimal control problems under uncertainty

for obtaining the educational and scientific degree

"DOCTOR"

In Scientific field 4. Sciences, matematics and informatics
Professional field 4.5 Mathematics
Doctoral program "Operations research"
Department PORS, FMI of Sofia University

Member of the jury: Prof. Tsvetomir Tsachev, PhD, IMI-BAS

The present referee's report is written on the basis of the version of the thesis which was officially submitted to the members of the scientific jury. The additional information provided to the members of the jury by the author of the thesis in two electronic messages (on April 30, 2024 and May 05, 2024 respectively) is also taken into account.

The submitted PhD thesis named "Optimal control problems under uncertainty" consists of 88 pages, divided into an introduction, three chapters, conclusion, appendix and bibliography of 53 references. It is written in English.

The results in the thesis are new and they constitute an original contribution to the scientific field they belong to. The references to previous results are comprehensive and correct. The presentation is straightforward. The Abstract adequately presents the results of the thesis.

The content of the chapters is as follows:

Linear-quadratic games with infinite time horizon are considered in Chapters 1 an 2, in Chapter 1 in continuous time (differential games), and in Chapter 2 – in discrete time. Nash equilibrium type issues are studied. In both cases (continuous or discrete time) the respective game in infinite time horizon (infinitely many steps in the discrete case) is approximated by a suitably defined game in finite time horizon (finitely many steps in the discrete case). The technique used in both chapters is based on solving a properly formulated matrix equation, the solution of which plays a crucial role in constructing the respective optimal feedback controls. The idea of the proofs consists in establishing the existence of a neighborhood of the origin in the state space where the constraints on the control of the first player are not binding and where the state trajectory will eventually fall.

The result of Aseev, Krastanov and Veliov of 2017 for an optimal control problem in discrete time and with infinitely many steps is extended for the case of zero-sum discrete dynamic game in Chapter 3. A necessary condition for Nash equilibrium in terms of a suitably defined Hamiltonian is obtained. A sufficient condition for optimality is proved for a discrete time optimal control problem with infinite time horizon (infinitely many steps).

Illustrative examples are presented – one in Chapter 2 and one in Chapter 3. I would like to point out the example from Chapter 2 related to modeling the short term longitudinal changes in an aircraft flight.

Remarks.

1. The saddle point inequalities (1.5) (in the claim of Proposition 1.2) are neither valid for arbitrary $\mathbf{v}$ with fixed $\bar{\mathbf{u}}$, nor for arbitrary $\mathbf{u}$ with fixed $\bar{\mathbf{v}}$, because when $\mathbf{v}$ is chosen, the $\bar{\mathbf{u}}$ defined in (1.4) depends on $\mathbf{v}$ via the state variable $\mathbf{x}$ (and when $\mathbf{u}$ is chosen, the $\bar{\mathbf{v}}$ defined in (1.4) depends on $\mathbf{u}$ via the state variable $\mathbf{x}$).

2. The same remark is valid for the proof of Theorem 2.5.

3. In an e-mail message of April 30, 2024 the author of the thesis informed the members of the jury that the claim of Theorem 2.1 is false because an error has been found in its proof. This theorem is used in the proof of Theorem 2.5, which is based on the approximation of discrete time game with infinitely many steps by the respective game with finitely many steps. In an e-mail message of May 05, 2024 the author of the thesis presented to the members of the jury a new version of the value of the game with finitely many steps which (game with finitely many steps) is used

for approximating the discrete time game with infinitely many steps. The newly defined value of game with finitely many steps eliminates the flaw, created by the error in the proof of Theorem 2.1, but now the value on the game with infinitely many steps has to be changed accordingly, so that the respective limiting prcess from the game with with finitely many steps to the game with infinitely many steps can be performed correctly.

4. The method (methods) used to solve the matrix equation (2.5) is (are) not described in the presentation of the example in §2.5 of Chapter 2.

The results of the thesis are published in 3 papers as follows:

– one, joint with the scientific advisor and R. Rozenov, published in Dynamic Games and Applications, IF 1.5 (2022);
– one, joint with the scientific advisor, published in the series Lecture Notes in Computer Science, Scopus 2022 SJR = 0.22;
– one, joint with the scientific advisor, accepted for publication (at the official submission of the thesis) in the series Lecture Notes in Computer Science, Scopus 2022 SJR = 0.22

Based on the above I recommend the Honorable scientific jury to award Boyan Kolev Stefanov the educational and scientific degree "Doctor" in Professional field 4.5 Mathematics, Scientific subject "Operations research" after he has comprehensively answered to the above mentioned remarks.

May 10, 2024 Prof. Tsvetomir Tsachev, PhD
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